

# Subteaming and Adaptive Formation Control for Coordinated Multi-Robot Navigation

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**Abstract:** Coordinated multi-robot navigation is essential for robots to operate as a team in diverse environments. During navigation, robot teams usually need to maintain specific formations, such as circular formations to protect human teammates at the center. However, in complex scenarios such as narrow corridors, rigidly preserving predefined formations can become infeasible. Therefore, robot teams must be capable of dynamically splitting into smaller subteams and adaptively controlling the subteams to navigate through such scenarios while preserving formations. To enable this capability, we introduce a novel method for *SubTeaming and Adaptive Formation* (STAF), which is built upon a unified hierarchical learning framework: (1) high-level deep graph cut for team splitting, (2) intermediate-level graph learning for facilitating coordinated navigation among subteams, and (3) low-level policy learning for controlling individual mobile robots to reach their goal positions while avoiding collisions. To evaluate STAF, we conducted extensive experiments in both indoor and outdoor environments using robotics simulations and physical robot teams. Experimental results show that STAF enables the novel capability for subteaming and adaptive formation control, and achieves promising performance in coordinated multi-robot navigation through challenging scenarios. More details are available on the project website: <https://anonymous188.github.io/STAF/>.

**Keywords:** Coordinated multi-robot navigation, subteam, hierarchical learning.

## 20 1 Introduction

21 Multi-robot systems have attracted growing at-  
 22 tention due to their advantages, such as redun-  
 23 dancy [1], parallelism [2], and scalability [3].  
 24 Coordinated multi-robot navigation is a funda-  
 25 mental capability that allows teams of robots to  
 26 traverse environments in a synchronized manner  
 27 and reach goal positions collectively [4]. This  
 28 capability is crucial in real-world applications,  
 29 such as search and rescue [5, 6, 7], space explo-  
 30 ration [8, 9], and transportation [10, 11].

31 During coordinated navigation, robots are often  
 32 required to maintain mission-specific formation,  
 33 such as circular formations for protection or line  
 34 formations for coverage. However, rigid adher-  
 35 ence to predefined formations can hinder effec-  
 36 tive navigation in certain scenarios. For instance, Figure 1 depicts a team of ten robots in a circular  
 37 formation encountering a corridor too narrow for the entire team to pass through. Thus, the team must  
 38 be capable of dynamically dividing into smaller units that operate both independently and cohesively

Figure 1: When a robot team in circular formation encounters a bridge that is too narrow for the entire team to cross at once. The robots must divide into subteams, adapt their formations to navigate the bridge, and rejoin the full team after crossing.

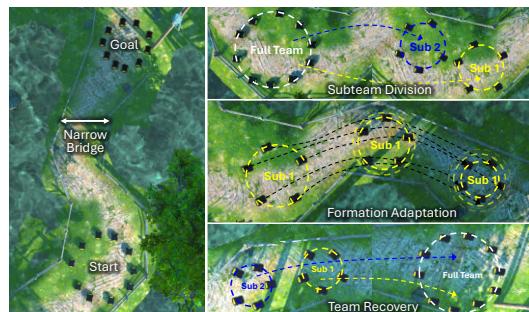


Figure 1: When a robot team in circular formation encounters a bridge that is too narrow for the entire team to cross at once. The robots must divide into subteams, adapt their formations to navigate the bridge, and rejoin the full team after crossing.

39 (i.e., *subteaming*) and controlling the subteams to pass through the narrow corridor while adaptively  
40 maintaining a specific formation (i.e., *adaptive formation control*).

41 The importance of coordinated multi-robot navigation has driven the development of various tech-  
42 niques. Traditional approaches, including classical planning methods [12], game-theoretic methods  
43 [13, 14], and optimization-based methods [15], often face high computational costs. Recently,  
44 learning-based methods like deep neural networks [16, 17] and multi-agent reinforcement learning  
45 [18, 19] have been used for modeling, coordination, and navigation. However, these methods have  
46 not addressed adaptive formation control, which is critical for narrow corridor traversal. Subteaming  
47 methods, such as graph cuts for team division [20, 21] and mixed-integer programming for task  
48 allocation [1, 22, 23], focus on team division alone and lack control over subteams or individual  
49 robots, which limits their effectiveness for coordinated navigation.

50 To address the challenges above and enable effective coordinated multi-robot navigation in complex  
51 scenarios where the entire robot team cannot pass through, we introduce a novel approach called  
52 *SubTeaming and Adaptive Formation (STAF)*, which offers new capabilities for subteam division,  
53 formation adaptation, and team recovery. Specifically, we design a graph representation to encode a  
54 team of robots, where each node represents a robot along with its associated attributes, such as its  
55 position, velocity, goal, and distance to obstacles, and each edge represents the spatial relationships  
56 between pairs of robots. Our STAF approach integrates three levels of robot learning into a hierarchical  
57 framework. At the high level, given the graph representation of a robot team, STAF performs deep  
58 graph cuts to divide the entire robot team into subteams. The intermediate level of STAF focuses  
59 on learning the coordination of these robot subteams for navigation, which develops a graph neural  
60 network with learnable message sharing to coordinate robots within a subteam, while generating  
61 graph embeddings to encode the subteam context. Finally, at the low level, given these embeddings,  
62 STAF employs reinforcement learning to learn a navigation policy that controls each individual robot  
63 to adaptively maintain subteam formation, reach the goal position, and avoid collisions.

64 Our primary contribution is the introduction of the novel STAF method to enable a new multi-robot  
65 navigation capability of subteaming and formation adaptation. The specific novelties include:

- 66 This work introduces one of the first problem formulations and learning-based solutions  
67 for subteaming and formation adaptation in multi-robot coordinated navigation. It enables  
68 new multi-robot capabilities, including subteam division, formation adaptation, and team  
69 recovery, allowing a team of robots to navigate complex environments in a coordinated  
70 manner, particularly narrow corridors where maintaining original formation is infeasible.
- 71 We introduce a novel hierarchical robot learning method that simultaneously integrates  
72 high-level deep graph cut for subteaming, intermediate-level graph learning for subteam  
73 coordination and adaptive formation control, and low-level individual robot control for  
74 collision-free navigation in complex environments.

## 75 2 Related Work

76 **Hierarchical Learning for Robotics** Hierarchical learning has shown promise in complex multi-  
77 robot tasks by providing a structured problem formulation that better aligns with multi-objective goals.  
78 It also enhances modularity in model design, which improves interpretability and enables clearer  
79 evaluation of each level. Applications include task allocation [24], maintaining communication [25],  
80 path planning [26, 27], and consensus reaching [28]. Typically, the lower level handles individual  
81 control tasks such as obstacle avoidance [29, 30]. The upper level focuses on team planning and  
82 coordination [31, 32, 33, 18]. However, applying hierarchical learning to formation adaptation  
83 and subteaming remains challenging due to the need for scalable team representations, dynamic  
84 adaptation, and efficient integration of formation control with flexible team reconfiguration.

85 **Coordinated Multi-Robot Navigation** Learning-free methods rely on predefined formation strate-  
86 gies, such as leader-follower [15, 4, 34, 35] and virtual region methods [36, 37, 38, 39]. However,  
87 these rigid formations lack adaptability to environmental changes. Learning-based methods, such

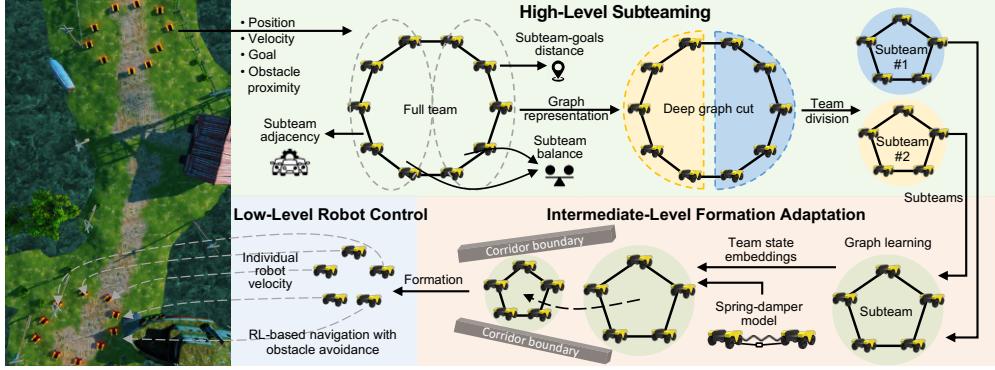


Figure 2: Overview of STAF, which integrates three levels of robot learning within a unified hierarchical learning framework to enable coordinated multi-robot navigation.

as reinforcement learning (RL) [40, 41, 18, 19, 42, 41], address this limitation by optimizing actions through environmental feedback. Graph neural networks (GNNs) enhance team coordination and communication [43, 17], enabling decentralized decision-making [16, 44]. These approaches have been applied in areas such as connected autonomous driving [45, 8], area coverage [46], and search-and-rescue missions [5]. However, none of these methods effectively address subteaming and formation adaptation in coordinated navigation, particularly in complex narrow corridors.

**Subteaming in Multi-Robot Navigation and Task Allocation** Subteaming increases the complexity of coordinated multi-robot navigation as it involves splitting, merging, and reformation based on tasks or environments. Graph-based methods [20, 21, 47, 1] use graph partitioning and cutting to determine team division and merging, but often rely on explicit connectivity constraints. Leader-follower methods [48, 49, 15] apply predefined hierarchy-based strategies but lack flexibility in dynamic environments. Optimization-based approaches [50, 22, 23, 51] compute optimal assignments via mixed-integer programming. Heuristic-based methods [52, 53] use problem-specific heuristics to determine team formation and coordination strategies. However, these methods focus on team division alone and lack control over subteams or individual robots. See Appendix A for details.

### 3 Approach

**Problem Definition** We discuss our STAF method that enables new multi-robot capabilities of subteaming and formation adaptation for coordinated multi-robot navigation. An overview of STAF is illustrated in Figure 2. We represent a team of  $n$  robots using an undirected graph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ . In the node set  $\mathcal{V} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ , each node  $\mathbf{v}_i = \{\mathbf{p}_i, \mathbf{g}_i, \mathbf{q}_i\}$  consists of the attributes of the  $i$ -th robot, where  $\mathbf{p}_i = [p_i^x, p_i^y]$  denotes its position,  $\mathbf{g}_i = [g_i^x, g_i^y]$  denotes its goal position, and  $\mathbf{q}_i = [q_i^x, q_i^y]$  denotes its velocities along x and y directions. The edge matrix  $\mathcal{E} = \{a_{i,j}\}_{n \times n}$  represents the spatial adjacency of the robots, where  $a_{i,j} = 1$ , if the  $i$ -th robot and the  $j$ -th robot are within a radius; otherwise  $a_{i,j} = 0$ . We further define the state of the  $i$ -th robot  $\mathbf{s}_i = [\mathbf{p}_i, \mathbf{g}_i, \mathbf{q}_i, c_i]$  as the concatenation of the robot's attributes and the distance  $c_i$  between the robot and its closest obstacle. We define the action of the  $i$ -th robot as  $\mathbf{a}_i = [v_i^x, v_i^y]$ , where  $v_i^x$  and  $v_i^y$  denote the robot's velocities in the x and y directions, respectively.

Our objective is to address the problems of subteaming and formation adaptation in the context of coordinated multi-robot navigation:

- **Formation Adaptation:** The capability of a robot team or subteam to maintain a desired formation while dynamically adjusting their relative positions to safely and efficiently navigate through the unstructured environment toward their goal positions, particularly in challenging scenarios such as narrow corridors.
- **Subteaming:** The capability of a robot team with a specific formation to autonomously divide into subteams with the same formation type when navigating environments too narrow

123 for the entire robot team. After successfully passing through, the subteams must merge back  
 124 into the full team, restoring the original formation.

125 **High-Level Deep Graph Cut for Subteaming** Given the graph  $\mathcal{G}$  as the representation of the robot  
 126 team, we introduce a new deep graph cut approach at the high level of STAF to enable subteaming.  
 127 We compute the embedding of the robot graph as  $\mathcal{H} = \{\mathbf{h}_i\} = \omega(\mathcal{G})$ , where  $\mathbf{h}_i$  is the embedding of  
 128 the  $i$ -th robot and  $\omega$  is a graph attention network [54]. We project each node into a representation  
 129 space by calculating  $\mathbf{m}_i = \mathbf{W}^v \mathbf{p}_i$ , where  $\mathbf{m}_i$  denotes the projected feature vector of the  $i$ -th node,  
 130 and  $\mathbf{W}^v$  denotes the weight matrix. Then, we compute the attention  $\alpha_{i,j}$  from the  $j$ -th node to  
 131 the  $i$ -th node as  $\alpha_{i,j} = \frac{\exp(\text{ReLU}([\mathbf{W}^a \mathbf{m}_i || \mathbf{W}^a \mathbf{m}_j]))}{\sum_{k \in \mathcal{N}(i)} \exp(\text{ReLU}([\mathbf{W}^a \mathbf{m}_i || \mathbf{W}^a \mathbf{m}_k]))}$ , where  $\text{ReLU}$  denotes the rectified linear  
 132 unit activation function,  $\mathcal{N}(i)$  represents the set of adjacent nodes of the  $i$ -th node,  $||$  denotes the  
 133 concatenation operation, and  $\mathbf{W}^a$  represents the weight matrix. The attention  $\alpha_{i,j}$  is obtained by  
 134 computing the similarity of the  $i$ -th node with its  $j$ -th adjacent nodes, followed by the SoftMax  
 135 normalization. Then, the final embedding  $\mathbf{h}_i$  for the  $i$ -th node is computed through aggregating  
 136 the embeddings of all its adjacent nodes as  $\mathbf{h}_i = \mathbf{W}^h \mathbf{m}_i + \sum_{j \in \mathcal{N}(i)} \alpha_{i,j} \mathbf{W}^h \mathbf{m}_j$ , where  $\mathbf{W}^h$  is the  
 137 weight matrix. We further utilize a multi-head mechanism [54] after the attention layers to enable the  
 138 network to capture a richer embedding representation.

139 Given  $\mathcal{H} = \{\mathbf{h}_i\}$ , we formulate subteaming as a graph cut problem, which partitions the entire  
 140 graph (representing the full team) into  $m$  subgraphs (representing subteams). In order to compute  
 141 team division, we develop a classifier network  $\tau(\mathcal{H})$  consisting of two fully connected linear layers  
 142 followed by a SoftMax function, which outputs the team division results as  $\mathbf{Y} = \tau(\mathcal{H}) = \{y_{i,j}\}^{n \times m}$ ,  
 143 where  $y_{i,j}$  is the probability of the  $i$ -th robot belonging to the  $j$ -th subteam, and  $m < n$ .

144 To ensure that robots within the same subteam group together, i.e., each robot is adjacent to its  
 145 teammates within the same subteam, we define a loss function that maximizes the adjacency of  
 146 robots within each subteam as  $\mathbf{Y}(1 - \mathbf{Y})^\top \mathbf{E}$ , where  $\mathbf{Y}(1 - \mathbf{Y})^\top$  calculates the probability that a  
 147 pair of robots belong to different subteams, and  $\mathbf{E}$  encodes the adjacency of the robots. In addition,  
 148 we aim to maintain balance in the sizes of robot subteams, encouraging each subteam to have  
 149 the same or a similar number of robots. It can be mathematically modeled by a loss function  
 150  $\sum_{j=1}^m (\sum_{i=1}^n y_{i,j} - \frac{n}{m})$ . The term  $\frac{n}{m}$  calculates the optimal size of balanced subteams (e.g., when  
 151  $n = 10$  and  $m = 2$ , each subteam would consist of 5 robots). Furthermore, we model the mission  
 152 objective of reaching the goal position by minimizing the overall distance between the subteams  
 153 and their respective goal positions. It can be mathematically defined as  $\sum_{j=1}^m \left\| \frac{\sum_{i=1}^n y_{i,j} \mathbf{p}_i}{\sum_{i=1}^n y_{i,j}} - \frac{\sum_{i=1}^n y_{i,j} \mathbf{g}_i}{\sum_{i=1}^n y_{i,j}} \right\|_2$ , where  $\frac{\sum_{i=1}^n y_{i,j} \mathbf{p}_i}{\sum_{i=1}^n y_{i,j}}$  denotes the center position of the  $j$ -th subteam and  $\frac{\sum_{i=1}^n y_{i,j} \mathbf{g}_i}{\sum_{i=1}^n y_{i,j}}$   
 155 denotes the center position of the goal for the subteam.

156 The high-level component of STAF performs an unsupervised graph cut to enable team division for  
 157 subteaming by minimizing the following objective function:

$$\mathcal{L}_{st} = \underbrace{\mathbf{Y}(1 - \mathbf{Y})^\top \mathbf{E}}_{\text{Subteam adjacency}} + \underbrace{\sum_{j=1}^m \left( \sum_{i=1}^n y_{i,j} - \frac{n}{m} \right)}_{\text{Subteam balance}} + \underbrace{\sum_{j=1}^m \left\| \frac{\sum_{i=1}^n y_{i,j} \mathbf{p}_i}{\sum_{i=1}^n y_{i,j}} - \frac{\sum_{i=1}^n y_{i,j} \mathbf{g}_i}{\sum_{i=1}^n y_{i,j}} \right\|_2}_{\text{Subteam-goals distance}} \quad (1)$$

158 which jointly accounts for subteam adjacency, subteam balance, and subteam-goal distances.

159 **Intermediate-Level Graph Learning for Multi-robot Formation Adaptation** To enable adaptive  
 160 multi-robot formation control, we develop a graph learning approach at the intermediate level of  
 161 STAF, which coordinates multiple robots to maintain a specific formation while adapting it based  
 162 on the surrounding environment. Given  $\mathcal{G}$  that represents a team (or subteam) of robots along with  
 163 the state  $\mathbf{s}_i$  for each robot  $i$ , we develop a graph network  $\phi$  to compute the embedding  $\mathbf{f}_i = \phi(\mathbf{s}_i, \mathcal{G})$   
 164 of the team state with respect to the  $i$ -th robot, which encodes the spatial relationships between the  
 165  $i$ -th robot with others in the team. The network  $\phi$  uses a linear layer to project the robot state  $\mathbf{s}_i$  to  
 166 the individual embedding  $\mathbf{z}_i$  of the  $i$ -th robot by  $\mathbf{z}_i = \mathbf{W}^z \mathbf{s}_i$ , where  $\mathbf{W}^z$  is the weight matrix of the  
 167 linear layer. Then, for the  $i$ -th robot,  $\phi$  aggregates individual state embeddings of all other teammates  
 168 through message passing to compute the team state embedding  $\mathbf{f}_i$  with respect to the  $i$ -th robot as

169  $\mathbf{f}_i = \mathbf{W}^f \mathbf{z}_i + \sum_{j \in \mathcal{N}(i)} \mathbf{W}^f (\mathbf{z}_j - \mathbf{z}_i)$ , where  $\mathbf{W}^f$  is the weight matrix. The team state embedding  
170  $\mathbf{f}_i$  with respect to the  $i$ -th robot encodes not only its own states (captured in the first term), but also  
171 the relative spatial relationships with other teammates (captured in the second term), which facilitates  
172 the coordination of actions to maintain specific formations during multi-robot navigation.

173 Robot teams and subteams may encounter scenarios, such as narrow corridors, where rigidly maintaining  
174 their formations prevents successful navigation. To enable formation adaptation, we implement  
175 a spring-damper model [55, 56] that dynamically adjusts the shape of the formation within the  
176 same type. This spring-damper model includes two components: (1) The spring component ensures  
177 that robot pairs maintain a balance between staying close enough to navigate narrow corridors and  
178 keeping a sufficient distance to prevent collisions, with the flexibility to adjust formation and enable  
179 adaptation. This spring component is modeled as  $|d_{i,j} - p_{i,j}|$ , where  $d_{i,j}$  denotes the expected  
180 distance in the original formation and  $p_{i,j}$  represents the actual distance between the  $i$ -th and  $j$ -th  
181 robots, computed as  $\|\mathbf{p}_i - \mathbf{p}_j\|_2$ . (2) The damper component prevents oscillation and overshooting  
182 of each robot during navigation by smoothing the relative velocities between pairs of robots, which is  
183 defined as  $q_{i,j} = \|\mathbf{q}_i - \mathbf{q}_j\|_2$ . Combining these components, the spring-damper model for formation  
184 adaptation is mathematically defined as  $R^{adp} = \sum_{\mathbf{v}_i, \mathbf{v}_j \in \mathcal{V}} -\lambda |d_{i,j} - p_{i,j}| - (1 - \lambda) q_{i,j}$ , where  $\lambda$   
185 is a hyperparameter that balances the importance of the spring and damper components.  $R^{adp}$  is  
186 incorporated into the reward function, which is used to derive a loss function for training STAF.

187 **Low-Level Individual Robot Control for Navigation** At the low-level of STAF, we introduce a  
188 navigation control network that outputs velocity commands as actions for each robot to reach its goal.  
189 Given the state  $\mathbf{s}_i$  for the  $i$ -th robot, we compute its state embedding  $\mathbf{f}_i$ . We design the network  $\psi$ ,  
190 which consists of two linear layers followed by the ReLU activation function, maps this embedding  
191 to an action as  $\mathbf{a}_i = \psi(\mathbf{f}_i)$ . The network  $\psi$  is a part of the control policy  $\pi_\theta(\mathbf{a}_i | \mathbf{s}_i)$ , parameterized by  
192  $\theta$ , which is trained using the framework of reinforcement learning. To enable each robot to move  
193 toward its target position and reach the navigational goal, we design a reward function based upon the  
194 distance between the current positions of the robot and its goal position. To enable obstacle avoidance  
195 for safe navigation, we implement a reward function that imposes a penalty when a robot comes too  
196 close to nearby obstacles or other robots in the team. When robots are divided into subteams, and  
197 once all subteams pass through the narrow corridor into an open area that is large enough for the full  
198 team, the goal position of each individual robot is updated to align with the full team’s goal, thereby  
199 recovering the subteams back into the full team with the original formation.

200 See Appendix B for details on **STAF Training and Execution** with their time complexity analysis.

## 201 4 Experiments

202 **Experimental Setups** We comprehensively evaluate our STAF approach across three setups: (1) a  
203 standard Gazebo simulation in ROS1, (2) a high-fidelity Unity-based 3D multi-robot simulator in  
204 ROS1, and (3) physical robot teams running ROS2. Each setup involves different numbers and types  
205 of robots arranged in formations such as circle, wedge, and line. In all scenarios, the environment  
206 includes narrow corridors, which require the full robot team to divide into subteams that adapt their  
207 formation to pass through. Afterward, the subteams regroup into the original full-team formation. In  
208 simulation, robot poses and obstacles are obtained from Gazebo and Unity. In real-world experiments,  
209 robots use a SLAM approach [57] for state estimation and mapping. See Appendix C for details on  
210 approach implementation and training. All video demonstrations are available on our project website.

211 We implement the complete STAF approach referred to as **STAF-full**. The full team divides into  
212 subteams to navigate through narrow environments, and after passing through, the subteams regroup  
213 into the full team to its original formation. To analyze the performance of the subteams, we refer  
214 to the subteams as **STAF-sub#**, e.g., STAF-sub1 and STAF-sub2. For comparison with STAF, we  
215 further implement two previous methods for multi-robot coordinated navigation, including: (1) A  
216 Leader and Follower method (**L&F**) [15] that one of the robots is designated as the “leader robot”  
217 that leads the movements of the other “follower robots” in the team while maintaining the formation.

Table 1: Quantitative comparison of STAF and Previous Methods from Gazebo simulations in ROS1.

Method	Circle Formation						Wedge Formation						Line Formation					
	SR (%)	TT (sec)	$\sigma < 0.5$	$\sigma < 0.1$	$\sigma < 0.01$	SR (%)	TT (sec)	$\sigma < 0.5$	$\sigma < 0.1$	$\sigma < 0.01$	SR (%)	TT (sec)	$\sigma < 0.5$	$\sigma < 0.1$	$\sigma < 0.01$			
DGNN [18]	<b>100.00</b>	68.70	60.41	58.91	58.91	<b>100.00</b>	82.70	47.85	42.33	41.92	<b>100.00</b>	72.61	27.90	20.16	20.16			
L&F [15]	40.00	<b>27.40</b>	67.28	64.54	62.69	70.00	<b>26.50</b>	69.70	62.11	59.47	60.00	<b>30.10</b>	63.76	55.89	55.89			
STAF-full	<b>100.00</b>	102.10	<b>87.79</b>	<b>80.12</b>	<b>80.12</b>	<b>100.00</b>	69.30	<b>80.52</b>	<b>80.51</b>	<b>80.50</b>	<b>100.00</b>	111.50	<b>91.45</b>	<b>80.06</b>	<b>78.93</b>			

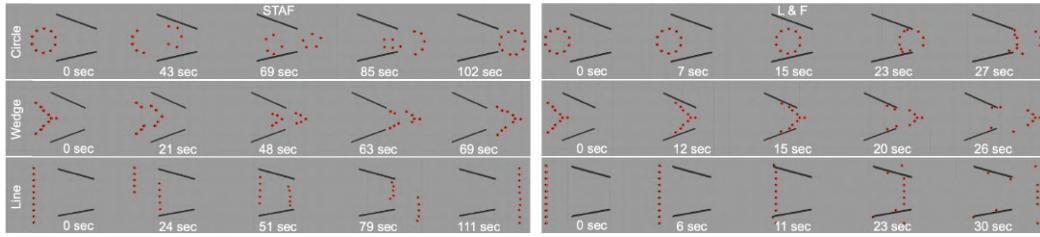


Figure 3: Qualitative results from Gazebo simulations on subteaming and formation adaptation.

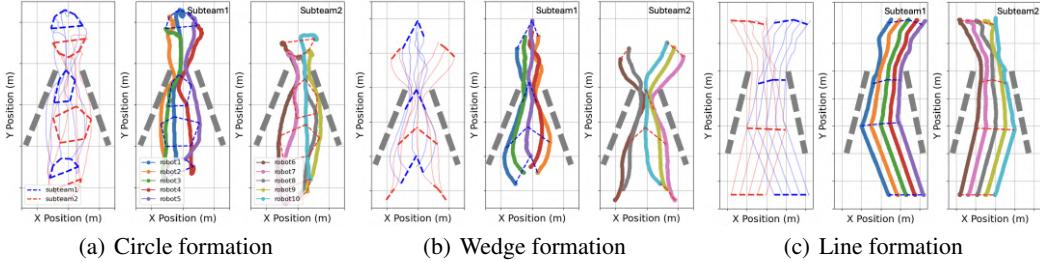


Figure 4: Movement trajectories of ten robots navigating a narrow corridor with different formations. In Figure 4(a) to 4(c), the first subfigure displays two subteams (red and blue) during team division, navigation with formation adaptation, and regrouping. The second and third subfigures show subteam trajectories, with each robot's path in a distinct color and gray dashed lines indicating obstacles.

218 (2) Decentralized GNN (**DGNN**) [18] that built upon a hierarchical learning framework to generate  
219 velocity controls for each individual robot for navigation, without considering team-level formations.

220 To quantitatively evaluate and compare with other methods, we employ three metrics, including:  
221 (1) Successful Rate (**SR**) is defined as the proportion of the robots within the full team that suc-  
222 cessfully reach goal positions without collisions. (2) Travel Time (**TT**) is defined as the total  
223 time used by the full team to reach the goal position. (3) Contextual Formation Integrity (**CFI**)  
224 is defined as the real-time adherence of the robots to their designated formation, given a shape  
225 threshold that defines the strictness of the formation. The CFI metric combines concepts of thresholds  
226 and uncertainty, which are commonly applied in computer vision [58]. It is formally defined as  
227  $w(1 - \sigma^{-1} \min(|r - (\eta + \sigma)|, |r - (\eta - \sigma)|)) + (1 - w)\epsilon$ . The  $CFI \in [0, 1]$  evaluates how effec-  
228 tively a robot team utilizes corridor gap and maintains formation. It combines two terms: the first  
229 measures spatial efficiency using the team's maximum radius  $r$ , corridor width  $\eta$ , and uncertainty  $\sigma$ ,  
230 where smaller  $\sigma$  indicates stricter formation requirements; the second  $\epsilon \in [0, 1]$  evaluates the integrity  
231 of the team shape. A weighting factor  $w$  balances the two terms, with higher CFI values indicating  
232 better performance. See Appendix D for details on CFI and its calculation of different formations.

233 **Results in Multi-Robot Simulations** The qualitative results in the Gazebo simulation are shown  
234 in Figure 3. L&F gets stuck in the narrow corridor due to the lack of subteaming and formation  
235 adaptation. In contrast, our method autonomously divides the team, enabling each subteam to adapt  
236 formations and reach the goal; the first subteam starts moving, followed by the second, and they  
237 eventually merge into the full formation. Notably, for wedge formations, team division prioritizes  
238 goal-distance objectives instead of maximizing connectivity, resulting in more compact subteams.

239 We visualize the trajectories of a team of 10 robots navigating in different formations, as shown in  
240 Figure 4. The visualization reveals subteaming behaviors (indicated by subteams in red and blue)  
241 colors), including team division and regrouping. Additionally, formation adaptation of each subteam  
242 occurs when navigating through narrow corridors (indicated by the individual robot trajectories).  
243 These results show the effectiveness of STAF in enabling both subteaming and formation adaptation.

244 The quantitative results are shown in Table 1. DGNN  
 245 performs the worst, particularly in the CFI metrics, as  
 246 it lacks formation control. L&F uses formation con-  
 247 trol and performs better but has only a 40% success  
 248 rate, as it lacks subteaming and formation adaptation,  
 249 which makes narrow corridor navigation difficult. Our  
 250 method outperforms both by addressing these limita-  
 251 tions, which achieves a 100% success rate. Although  
 252 STAF yields slightly longer travel times, this is expected due to its more complex navigation strategy.  
 253 Subteam performance in Table 2 shows a 100% success rate across all formations. STAF maintains  
 254 formation integrity above 87%, 80%, and 91% under the threshold  $\sigma < 0.5$  for circle, wedge, and line  
 255 formations, respectively. These results highlight the effectiveness of STAF in enabling coordinated  
 256 navigation through subteaming and adaptive formation control in complex environments.

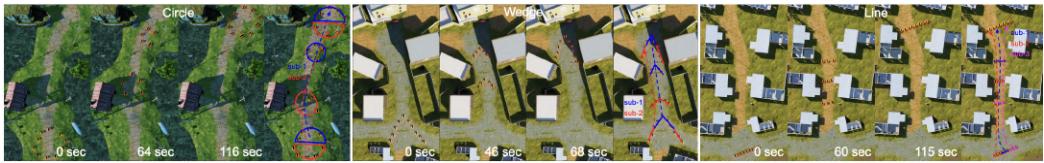


Figure 5: Qualitative results from Unity3D simulations in ROS1 using varying numbers of differential-drive Warthog robots in three formations while navigating a long, unstructured field environment.



Figure 6: Qualitative results from real-world experiments in both indoor narrow spaces and outdoor uneven terrain, using varying numbers of Limo robots running ROS2 and communicating via Wi-Fi.

257 Beyond the Gazebo simulation, we further use a high-fidelity Unity3D simulator in ROS1, which  
 258 simulates outdoor field environments with narrow pathways and bridges. Instead of using holonomic  
 259 robots as shown in the Gazebo simulation, we use differential-drive Warthog robots and convert  
 260 the linear velocity in the action  $a_i$  into wheel velocities to follow the same trajectory. This setting  
 261 introduces new challenges, which require the robot team to navigate complex, long curved paths  
 262 that demand continuous formation adjustments, as well as extremely narrow areas that demand  
 263 division into more than two subteams. As illustrated in Figures 5, our STAF approach successfully  
 264 addresses these challenges by dividing a full team into subteams, adapting actions of differential-drive  
 265 subteams to navigate, and regroup after subteam traversal. For line formation with 9 robots, STAF  
 266 can dynamically divide into three subteams to navigate a corridor too narrow for groups larger than 3.

267 **Case Study on Physical Robot Teams** We validate STAF on real-world case studies using differential-  
 268 drive Limo robots with caterpillar tracks, each equipped with an onboard Intel NCU i7 and running  
 269 ROS2 with Wi-Fi-based team communication. The real-world experiments are conducted both  
 270 indoors and outdoors, as shown in Figure 6. Our method enables teams of 6 to 8 robots to divide  
 271 into subteams and adapt formations to smoothly navigate narrow indoor spaces, including doorways,  
 272 hallways, and exits. In outdoor experiments on unstructured terrain such as passages between bollards,  
 273 scattered trees, and roadblocks, the results demonstrate the strong adaptability of our approach to  
 274 unknown environments. Subteaming and formation adaptation are effectively performed even on  
 275 snowy and uneven terrain, where wheel slippage introduces significant action uncertainty. Additional  
 276 Unity3D and real-world qualitative results with more timesteps are provided in the Appendix E.

## 277 5 Discussion

278 **Ablation Study on Subteam Division** We conduct an ablation study to evaluate the role of each  
 279 component in the objective function defined in Eq. (1) for team division. Figure 7(a) shows that

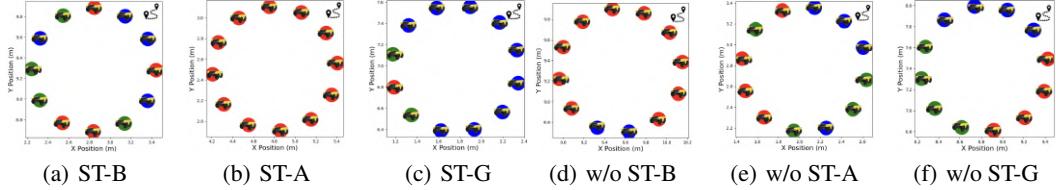


Figure 7: Ablation study that analyzes the impact of subteam division components: subteam balance (ST-B), subteam adjacency (ST-A), and subteam-goals distance (ST-G).

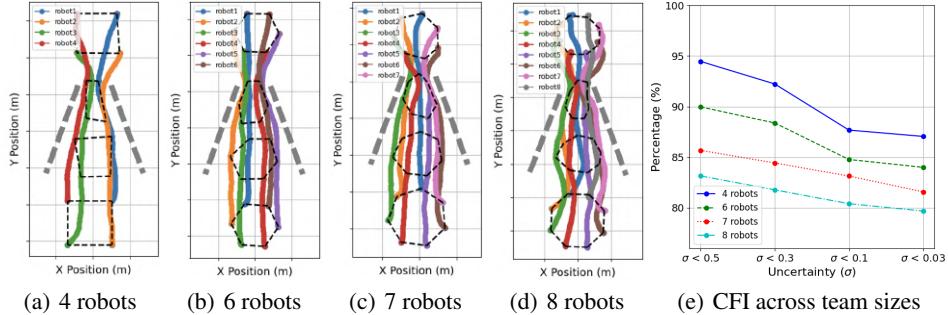


Figure 8: Quantitative results indicate STAF’s generalizability to different team sizes. Figures (a)-(d) show the trajectories of 4 to 8 robots in circle formations to navigate a narrow corridor. Figure (e) presents the variation in CFI values across different team sizes and  $\sigma$  values.

optimizing only the balance term evenly splits 12 robots into 3 subteams. Figure 7(b) shows that only maximizing adjacency leads to all robots being assigned to the same subteam. Figure 7(c) shows that only minimizing the goal-distance aligns subteams toward their goals (in the upper right). In addition, we remove each component individually to assess its impact. Figure 7(d) shows unbalanced team division without the balance term. Figure 7(e) results in uncompact subteams without the adjacency term. Figure 7(f) shows subteams misaligned with goals, which leads to inefficient navigation. These results further indicate the effectiveness and importance of enforcing subteam balance, maximizing adjacency, and minimizing subteam-goals distance for robot team division.

**Generalizability to Different Team Sizes** We evaluate the generalizability of STAF to different team sizes by varying the number of robots. Figures 8(a)-8(d) present the qualitative results on formation adaptation for teams of 4, 6, 7, and 8 robots in circle formation, which validate STAF’s generalizability across team sizes. Figure 8(e) presents the quantitative results using the CFI metric, which shows 87% formation integrity for 4 robots under  $\sigma < 0.03$ , and at least 80% for 8 robots.

**Generalizability to Different Numbers of Sub-teams** We evaluate STAF’s generalizability in dividing the team into varying numbers of subteams. As shown in Figure 9, STAF effectively handles divisions into 2, 3, and 4 subteams. Figure 5 contains a scenario where a nine-robot line formation splits into three subteams to navigate a corridor too narrow for groups larger than four.

See Appendix F for STAF’s **Robustness to Noise** and **Applicability to Different Robot Platforms**.

## 6 Conclusion

In this paper, we propose STAF for coordinated multi-robot navigation in complex scenarios. STAF is built upon a unified hierarchical learning framework, including a high-level deep graph cut for dynamic team division, an intermediate-level graph learning for team coordination with adaptive formation control, and a low-level RL policy for individual robot control. Results from comprehensive experiments show that STAF enables new multi-robot capabilities for subteaming and formation adaptation, and significantly outperforms existing methods on coordinated multi-robot navigation.

309 **7 Limitations**

310 Our approach presents several limitations that suggest directions for future research. First, although  
311 STAF’s intermediate and low levels are executed in a decentralized fashion, STAF’s high level for  
312 team division is executed in a centralized fashion. One direction for future research is to decentralize  
313 the high-level team division, such as by replacing the current global graph cut optimization with  
314 a distributed consensus algorithm (e.g., gossip [59] or max-consensus [60]). These decentralized  
315 methods would enable each robot to determine its subteam based upon the information shared by its  
316 teammates through broadcasting, and iteratively reach a consensus and converge to a stable subteam  
317 assignment through negotiation. Second, the alternating training algorithm we use, which iteratively  
318 trains the high-level and joint intermediate-low levels, is considered a limitation, as it may lead  
319 to suboptimal integration of these levels and difficulties with training error propagation. In the  
320 future, we plan to integrate the high-level graph cut together with the joint intermediate-low level  
321 training into an end-to-end training algorithm, where the training error from the low level will be  
322 propagated not only to the intermediate level but also to the high level, which enables updates to the  
323 network parameters across all three levels. To achieve this, we will adopt a centralized training with  
324 decentralized execution strategy, where all levels of the hierarchy can leverage global information  
325 during training, while ensuring decentralized execution during deployment. The third limitation  
326 is that the number of subteams, as a hyperparameter, is decided manually. A future direction is to  
327 dynamically and adaptively determine this hyperparameter by selecting the minimum number of  
328 subteams such that the smallest formation of each subteam can successfully navigate through the  
329 narrowest corridor in the environment. The width of a corridor can be identified either by analyzing  
330 the environment map (using a prior map or built by a SLAM method) or through real-time robotic  
331 sensing.

332 **References**

333 [1] P. Gao, S. Siva, A. Micciche, and H. Zhang. Collaborative scheduling with adaptation to failure  
334 for heterogeneous robot teams. In *IEEE International Conference on Robotics and Automation*,  
335 2023.

336 [2] C. Pincioli, V. Trianni, R. O’Grady, G. Pini, A. Brutschy, M. Brambilla, N. Mathews, E. Fer-  
337 rante, G. Di Caro, F. Ducatelle, et al. Argos: A modular, parallel, multi-engine simulator for  
338 multi-robot systems. *Swarm Intelligence*, 6:271–295, 2012.

339 [3] T. Balch and M. Hybinette. Social potentials for scalable multi-robot formations. In *IEEE*  
340 *International Conference on Robotics and Automation*, 2000.

341 [4] P. Singh, R. Tiwari, and M. Bhattacharya. Navigation in multi robot system using cooperative  
342 learning: A survey. In *International Conference on Computational Techniques in Information*  
343 *and Communication Technologies*, 2016.

344 [5] J. P. Queralta, J. Taipalmaa, B. C. Pullinen, V. K. Sarker, T. N. Gia, H. Tenhunen, M. Gabbouj,  
345 J. Raitoharju, and T. Westerlund. Collaborative multi-robot search and rescue: Planning,  
346 coordination, perception, and active vision. *IEEE Access*, 8:191617–191643, 2020.

347 [6] Q. Yang and R. Parasuraman. Needs-driven heterogeneous multi-robot cooperation in rescue  
348 missions. In *IEEE International Symposium on Safety, Security, and Rescue Robotics*, 2020.

349 [7] J. L. Baxter, E. Burke, J. M. Garibaldi, and M. Norman. Multi-robot search and rescue: A  
350 potential field based approach. *Autonomous Robots and Agents*, pages 9–16, 2007.

351 [8] R. Han, S. Chen, and Q. Hao. Cooperative multi-robot navigation in dynamic environment with  
352 deep reinforcement learning. In *IEEE International Conference on Robotics and Automation*,  
353 2020.

354 [9] V. Indelman. Cooperative multi-robot belief space planning for autonomous navigation in  
355 unknown environments. *Autonomous Robots*, 42:353–373, 2018.

356 [10] A. Amanatiadis, C. Henschel, B. Birkicht, B. Andel, K. Charalampous, I. Kostavelis, R. May,  
 357 and A. Gasteratos. Avert: An autonomous multi-robot system for vehicle extraction and  
 358 transportation. In *IEEE International Conference on Robotics and Automation*, 2015.

359 [11] D. Koung, O. Kermorgant, I. Fantoni, and L. Belouaer. Cooperative multi-robot object trans-  
 360 portation system based on hierarchical quadratic programming. *IEEE Robotics and Automation*  
 361 *Letters*, 6(4):6466–6472, 2021.

362 [12] J. J. Kuffner and S. M. LaValle. Rrt-connect: An efficient approach to single-query path  
 363 planning. In *IEEE International Conference on Robotics and Automation*, 2000.

364 [13] B. Tang, K. Xiang, M. Pang, and Z. Zhanxia. Multi-robot path planning using an improved  
 365 self-adaptive particle swarm optimization. *International Journal of Advanced Robotic Systems*,  
 366 17(5):1729881420936154, 2020.

367 [14] D. Cappello, S. Garcin, Z. Mao, M. Sassano, A. Paranjape, and T. Mylvaganam. A hybrid  
 368 controller for multi-agent collision avoidance via a differential game formulation. *IEEE*  
 369 *Transactions on Control Systems Technology*, 29(4):1750–1757, 2021.

370 [15] B. Reily, C. Reardon, and H. Zhang. Leading multi-agent teams to multiple goals while  
 371 maintaining communication. In *Robotics Science and Systems*, 2020.

372 [16] M. Goarin and G. Loianno. Graph neural network for decentralized multi-robot goal assignment.  
 373 *IEEE Robotics and Automation Letters*, 2024.

374 [17] S. Zhang, K. Garg, and C. Fan. Neural graph control barrier functions guided distributed  
 375 collision-avoidance multi-agent control. In *Conference on Robot Learning*, 2023.

376 [18] J. Blumenkamp, S. Morad, J. Gielis, Q. Li, and A. Prorok. A framework for real-world multi-  
 377 robot systems running decentralized GNN-based policies. In *International Conference on*  
 378 *Robotics and Automation*, 2022.

379 [19] Y. Hu, J. Fu, and G. Wen. Graph soft actor-critic reinforcement learning for large-scale  
 380 distributed multirobot coordination. *IEEE Transactions on Neural Networks and Learning*  
 381 *Systems*, 2023.

382 [20] H. Zhu, J. Juhl, L. Ferranti, and J. Alonso-Mora. Distributed multi-robot formation splitting and  
 383 merging in dynamic environments. In *International Conference on Robotics and Automation*,  
 384 2019.

385 [21] B. Reily, C. Reardon, and H. Zhang. Representing multi-robot structure through multimodal  
 386 graph embedding for the selection of robot teams. In *IEEE International Conference on Robotics*  
 387 *and Automation*, 2020.

388 [22] W. J. Jose and H. Zhang. Learning for dynamic subteaming and voluntary waiting in heteroge-  
 389 neous multi-robot collaborative scheduling. In *IEEE International Conference on Robotics and*  
 390 *Automation*, 2024.

391 [23] G. A. Cardona, K. Leahy, and C.-I. Vasile. Temporal logic swarm control with splitting and  
 392 merging. In *IEEE International Conference on Robotics and Automation*, 2023.

393 [24] R. Wang, Z. Hua, G. Liu, J. Zhang, J. Yan, F. Qi, S. Yang, J. Zhou, and X. Yang. A bi-level  
 394 framework for learning to solve combinatorial optimization on graphs. *Advances in Neural*  
 395 *Information Processing Systems*, 34:21453–21466, 2021.

396 [25] M. Bettini, A. Shankar, and A. Prorok. Heterogeneous multi-robot reinforcement learning.  
 397 *arXiv*, 2023.

398 [26] J. Zhang, J. Ge, S. Li, S. Li, and L. Li. A bi-level network-wide cooperative driving approach  
 399 including deep reinforcement learning-based routing. *IEEE Transactions on Intelligent Vehicles*,  
 400 2023.

401 [27] W. Wang, L. Mao, R. Wang, and B.-C. Min. Multi-robot cooperative socially-aware navigation  
 402 using multi-agent reinforcement learning. In *IEEE International Conference on Robotics and*  
 403 *Automation*, 2024.

404 [28] P. Feng, J. Liang, S. Wang, X. Yu, X. Ji, Y. Chen, K. Zhang, R. Shi, and W. Wu. Hierarchical  
 405 consensus-based multi-agent reinforcement learning for multi-robot cooperation tasks. In  
 406 *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2024.

407 [29] B. Bischoff, D. Nguyen-Tuong, I. Lee, F. Streichert, A. Knoll, et al. Hierarchical reinforcement  
 408 learning for robot navigation. In *Proceedings of The European Symposium on Artificial Neural*  
 409 *Networks, Computational Intelligence And Machine Learning*, 2013.

410 [30] Y. Jin, S. Wei, J. Yuan, and X. Zhang. Hierarchical and stable multiagent reinforcement learning  
 411 for cooperative navigation control. *IEEE Transactions on Neural Networks and Learning*  
 412 *Systems*, 34(1):90–103, 2021.

413 [31] J. Wöhlke, F. Schmitt, and H. van Hoof. Hierarchies of planning and reinforcement learning for  
 414 robot navigation. In *IEEE International Conference on Robotics and Automation*, 2021.

415 [32] J. Hu, H. Niu, J. Carrasco, B. Lennox, and F. Arvin. Voronoi-based multi-robot autonomous  
 416 exploration in unknown environments via deep reinforcement learning. *IEEE Transactions on*  
 417 *Vehicular Technology*, 69(12):14413–14423, 2020.

418 [33] W. Zhu and M. Hayashibe. A hierarchical deep reinforcement learning framework with high  
 419 efficiency and generalization for fast and safe navigation. *IEEE Transactions on Industrial*  
 420 *Electronics*, 70(5):4962–4971, 2022.

421 [34] T. Wu, K. Xue, and P. Wang. Leader-follower formation control of usvs using APF-based  
 422 adaptive fuzzy logic nonsingular terminal sliding mode control method. *Journal of Mechanical*  
 423 *Science and Technology*, 36(4):2007–2018, 2022.

424 [35] H. Xiao and C. P. Chen. Leader-follower consensus multi-robot formation control using  
 425 neurodynamic-optimization-based nonlinear model predictive control. *IEEE Access*, 7:43581–  
 426 43590, 2019.

427 [36] D. Roy, A. Chowdhury, M. Maitra, and S. Bhattacharya. Multi-robot virtual structure switch-  
 428 ing and formation changing strategy in an unknown occluded environment. In *IEEE/RSJ*  
 429 *International Conference on Intelligent Robots and Systems*, 2018.

430 [37] N. Abujabal, R. Fareh, S. Sinan, M. Baziyad, and M. Bettayeb. A comprehensive review of  
 431 the latest path planning developments for multi-robot formation systems. *Robotica*, 41(7):  
 432 2079–2104, 2023.

433 [38] D. Roy, A. Chowdhury, M. Maitra, and S. Bhattacharya. Virtual region based multi-robot path  
 434 planning in an unknown occluded environment. In *IEEE/RSJ International Conference on*  
 435 *Intelligent Robots and Systems*, 2019.

436 [39] J. Alonso-Mora, E. Montijano, T. Nägeli, O. Hilliges, M. Schwager, and D. Rus. Distributed  
 437 multi-robot formation control in dynamic environments. *Autonomous Robots*, 43:1079–1100,  
 438 2019.

439 [40] R. Han, S. Chen, S. Wang, Z. Zhang, R. Gao, Q. Hao, and J. Pan. Reinforcement learned  
 440 distributed multi-robot navigation with reciprocal velocity obstacle shaped rewards. *IEEE*  
 441 *Robotics and Automation Letters*, 7(3):5896–5903, 2022.

442 [41] N. Hacene and B. Mendil. Behavior-based autonomous navigation and formation control  
443 of mobile robots in unknown cluttered dynamic environments with dynamic target tracking.  
444 *International Journal of Automation and Computing*, 18(5):766–786, 2021.

445 [42] R. Han, S. Chen, and Q. Hao. Cooperative multi-robot navigation in dynamic environment with  
446 deep reinforcement learning. In *IEEE International Conference on Robotics and Automation*,  
447 2020.

448 [43] Q. Li, F. Gama, A. Ribeiro, and A. Prorok. Graph neural networks for decentralized multi-robot  
449 path planning. In *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2020.

450 [44] Z. Gao, G. Yang, and A. Prorok. Co-optimization of environment and policies for decentralized  
451 multi-agent navigation. *arXiv*, 2024.

452 [45] P. Gao, Y. Shen, and M. C. Lin. Collaborative decision-making using spatiotemporal graphs in  
453 connected autonomy. *IEEE International Conference on Robotics and Automation*, 2024.

454 [46] Ö. Özkahraman and P. Ögren. Collaborative navigation-aware coverage in feature-poor environ-  
455 ments. In *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2022.

456 [47] W. Luo, S. Yi, and K. Sycara. Behavior mixing with minimum global and subgroup connectivity  
457 maintenance for large-scale multi-robot systems. In *IEEE International Conference on Robotics  
458 and Automation*, 2020.

459 [48] D. Roy, M. Maitra, and S. Bhattacharya. Exploration of multiple unknown areas by swarm of  
460 robots utilizing virtual-region-based splitting and merging technique. *IEEE Transactions on  
461 Automation Science and Engineering*, 19(4):3459–3470, 2021.

462 [49] S. Swaminathan, M. Phillips, and M. Likhachev. Planning for multi-agent teams with leader  
463 switching. In *IEEE International Conference on Robotics and Automation*, 2015.

464 [50] S. Novoth, Q. Zhang, K. Ji, and D. Yu. Distributed formation control for multi-vehicle systems  
465 with splitting and merging capability. *IEEE Control Systems Letters*, 5(1):355–360, 2020.

466 [51] Á. Calvo and J. Capitán. Optimal task allocation for heterogeneous multi-robot teams with  
467 battery constraints. In *IEEE International Conference on Robotics and Automation*, 2024.

468 [52] T. Guo, S. D. Han, and J. Yu. Spatial and temporal splitting heuristics for multi-robot motion  
469 planning. In *IEEE International Conference on Robotics and Automation*, 2021.

470 [53] T. Guo and J. Yu. Efficient heuristics for multi-robot path planning in crowded environments.  
471 In *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2023.

472 [54] P. Veličković, G. Cucurull, A. Casanova, A. Romero, P. Lio, and Y. Bengio. Graph attention  
473 networks. *International Conference on Representation Learning*, 2017.

474 [55] Z. Deng, P. Gao, W. J. Jose, and H. Zhang. Multi-robot collaborative navigation with formation  
475 adaptation. *arXiv*, 2024.

476 [56] C. Gabellieri, A. Palleschi, and L. Pallottino. Force-based formation control of omnidirectional  
477 ground vehicles. In *IEEE/RSJ International Conference on Intelligent Robots and Systems*,  
478 2021.

479 [57] Q. Zou, Q. Sun, L. Chen, B. Nie, and Q. Li. A comparative analysis of LiDAR SLAM-based  
480 indoor navigation for autonomous vehicles. *IEEE Transactions on Intelligent Transportation  
481 Systems*, 23(7):6907–6921, 2021.

482 [58] C. Godard, O. Mac Aodha, M. Firman, and G. J. Brostow. Digging into self-supervised  
483 monocular depth estimation. In *Proceedings of the IEEE/CVF International Conference on  
484 Computer Vision*, 2019.

485 [59] S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah. Randomized gossip algorithms. *IEEE*  
486 *Transactions on Information Theory*, 52(6):2508–2530, 2006.

487 [60] R. Olfati-Saber, J. A. Fax, and R. M. Murray. Consensus and cooperation in networked  
488 multi-agent systems. *Proceedings of the IEEE*, 95(1):215–233, 2007.

489 [61] E. Galceran and M. Carreras. A survey on coverage path planning for robotics. *Robotics and*  
490 *Autonomous Systems*, 61(12):1258–1276, 2013.

491 [62] R. Bohlin and L. E. Kavraki. Path planning using lazy PRM. In *IEEE International Conference*  
492 *on Robotics and Automation*, 2000.

493 [63] J. Alonso-Mora, S. Baker, and D. Rus. Multi-robot formation control and object transport in  
494 dynamic environments via constrained optimization. *The International Journal of Robotics*  
495 *Research*, 36(9):1000–1021, 2017.

496 [64] J. Alonso-Mora, E. Montijano, M. Schwager, and D. Rus. Distributed multi-robot formation  
497 control among obstacles: A geometric and optimization approach with consensus. In *IEEE*  
498 *International Conference on Robotics and Automation*, 2016.

499 [65] B. Reily, T. Mott, and H. Zhang. Adaptation to team composition changes for heterogeneous  
500 multi-robot sensor coverage. In *IEEE International Conference on Robotics and Automation*,  
501 2021.

502 [66] D. Koung, O. Kermorgant, I. Fantoni, and L. Belouaer. Cooperative multi-robot object trans-  
503 portation system based on hierarchical quadratic programming. *IEEE Robotics and Automation*  
504 *Letters*, 6(4):6466–6472, 2021.

505 [67] J. Schulman, F. Wolski, P. Dhariwal, A. Radford, and O. Klimov. Proximal policy optimization  
506 algorithms. *arXiv*, 2017.

507 [68] S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein, et al. Distributed optimization and statistical  
508 learning via the alternating direction method of multipliers. *Foundations and Trends® in*  
509 *Machine Learning*, 3(1):1–122, 2011.

510 [69] T. D. Barfoot. *State estimation for robotics*. Cambridge University Press, 2024.

511 **Appendix**

512 **A Extended Related Work**

513 In this section, we will first review existing techniques for learning-free coordinated multi-robot  
514 navigation. Then, we will review previous methods for subteaming in the areas of multi-robot  
515 navigation and task allocation. Finally, we present the state-of-the-art works on hierarchical learning  
516 for robotics.

517 **A.1 Learning-Free Coordinated Multi-Robot Navigation**

518 We review existing methods from two main perspectives: the multi-robot team formation and the  
519 theoretical perspective to enhance coordinated efficiency. From the multi-robot team formation angle,  
520 prevalent configurations include the leader-follower structure, where follower agents are programmed  
521 to maintain group behavior by following a leader agent [15, 4, 34, 35]. Additionally, virtual region  
522 methods that allow robot teams to adjust their formation within specified virtual areas are thoroughly  
523 explored [36, 37, 38, 39]. However, these formations are often rigid and lack the flexibility needed to  
524 adapt to complex environments that require dynamic formation changes.

525 From the theoretical perspective, classic methods of coordinated multi-robot navigation are catego-  
526 rized into three groups: traditional planning methods, game-theoretical approaches, and optimization-  
527 based techniques. Traditional planning methods include algorithms, such as A\* and its variants  
528 [61], rapidly exploring random trees (RRT) [12], and probabilistic roadmap (PRM) [62]. Game-  
529 theoretical approaches model multi-robot navigation as cooperative games for path planning [13, 14].  
530 Optimization-based methods aim to optimize various objectives in order to coordinate multiple  
531 robots during navigation, such as identifying traversable areas to prevent collisions [63, 64], main-  
532 taining communication [15], maximizing area coverage [65], and addressing hierarchical quadratic  
533 programming (HQP) problems for cooperative tasks [11, 66]. Traditional methods in coordinated  
534 navigation are primarily based upon heuristic searching and typically incur substantial computational  
535 costs. Additionally, none of these previous classic methods effectively address subteaming and  
536 formation adaptation in the context of coordinated navigation, particularly in complex scenarios such  
537 as traversing narrow corridors.

538 **A.2 Subteaming in Multi-Robot Navigation and Task Allocation**

539 Integrating subteaming with coordinated multi-robot navigation introduces additional complexity be-  
540 yond the standard multi-robot coordination, which requires team splitting, merging, and reformation in  
541 response to environmental and task constraints. Existing methods can be broadly categorized into four  
542 groups, including graph-based, leader-follower-based, optimization-based, and heuristic-based meth-  
543 ods. Graph-based methods [20, 21, 47, 1] use graph partitioning and graph cut techniques to determine  
544 how to divide and merge teams, typically relying on explicit connectivity constraints. Leader-follower  
545 methods [48, 49, 15] employ predefined hierarchy-based motion strategies, where a subset of agents  
546 leads and others follow, limiting flexibility in dynamic environments. Optimization-based methods  
547 typically use mixed-integer programming [50, 22, 23, 51] to compute optimal assignments and motion  
548 plans. Heuristic-based methods [52, 53] offer computationally efficient alternatives by leveraging  
549 problem-specific heuristics to determine team formation and coordination strategies. However, these  
550 methods generally focus on team division alone, without the capability of controlling the subteams or  
551 individual robots, which makes them unsuitable for addressing coordinated navigation.

552 **A.3 Hierarchical Learning for Robotics**

553 Recently, learning-based methods have gained significant attention for improving coordinated navi-  
554 gation in multi-robot systems. Reinforcement learning (RL) approaches have shown promising results in  
555 enabling robots to adapt to environmental changes [40, 41]. However, single-level RL methods often  
556 struggle with convergence in complex scenarios. Graph neural networks (GNNs) have been used to

557 enhance team coordination and communication [43, 17], supporting decentralized decision-making  
 558 [16, 44]. Multi-agent reinforcement learning (MARL) further improves coordinated navigation by  
 559 training robots to cooperate effectively [18, 19, 42, 41]. These learning-based approaches have been  
 560 successfully applied in areas such as connected autonomous driving [45, 8], area coverage [46], and  
 561 search-and-rescue missions [5].

562 Hierarchical learning attracts increasing attention to address this issue for complex multi-robot  
 563 tasks, such as solving combinatorial optimization for multi-robot task allocation [24], maintaining  
 564 communication that ensures connectivity among robots [25], multi-robot path planning [26, 27] and  
 565 consensus reaching [28]. Specifically, the lower level policy aims to optimize individual robot control,  
 566 such as enabling obstacle avoidance [29, 30]. The upper level focuses on multi-robot planning and  
 567 coordination, such as selecting sub-goals through goal-based planning [31], dividing exploration  
 568 areas using dynamic Voronoi partitions [32], facilitating obstacle avoidance [33] and communication  
 569 between robots [18].

570 These methods leverage hierarchical policy to optimize both high-level task planning and low-level  
 571 motion control simultaneously, which achieves promising performance compared to traditional  
 572 methods that rely on predefined rules and explicit environment representations. However, applying  
 573 hierarchical RL to formation adaptation and subteaming remains an open challenge due to the need  
 574 for scalable representations of team structures, dynamic adaptation to changing environments, and  
 575 efficient integration of formation control with flexible team reconfiguration.

## 576 B STAF Training and Execution

### 577 B.1 STAF Training

578 To train STAF as a three-level hierarchical learning model, we design an alternating training algo-  
 579 rithm that iterates between using unsupervised learning to train the high level for sub-teaming and  
 580 using Proximal Policy Optimization (PPO) [67] to jointly train the intermediate level for formation  
 581 adaptation and the low level for individual robot control.

582 Specifically, the high-level training receives a 2D occupancy map of the environment (e.g., built  
 583 using a SLAM approach [57]), as well as the starting and goal positions of the robots within the  
 584 map as input. The high level is trained using ADMM as the optimization solver [68] by deriving  
 585 the gradient of the unsupervised loss function in Eq. (1) to update the weights  $\mathbf{W}^a$  and  $\mathbf{W}^h$  of the  
 586 deep graph cut network  $\tau$  for subteam division, which considers subteam adjacency, subteam balance,  
 587 and subteam-goal distance. In the same iteration, we fix the high-level model once its training is  
 588 complete, and we utilize PPO to jointly train the intermediate and low levels of STAF. We design the  
 589 overall reward as a weighted summation of the coordination reward in STAF’s intermediate level for  
 590 formation adaptation, and the navigation reward and obstacle avoidance reward for individual robot  
 591 control. These rewards are used to compute the advantage function  $A^{\pi_{old}}(\mathbf{s}_i, \mathbf{a}_i)$ , which quantifies  
 592 how much better taking action  $\mathbf{a}_i$  in state  $\mathbf{s}_i$  is compared to the old policy  $\pi_{\theta_{old}}(\mathbf{s}_i, \mathbf{a}_i)$ . Then, a loss  
 593 value is computed by aggregating the differences for all robots between the output of the updated PPO  
 594 policy  $\pi_{\theta}(\mathbf{s}_i, \mathbf{a}_i)$  and the old policy  $\pi_{\theta_{old}}(\mathbf{s}_i, \mathbf{a}_i)$ . To prevent instability in training due to large policy  
 595 updates, a clipping function  $\text{clip}(1 - \delta, 1 + \delta)$  is used to constrain the ratio between the updated and  
 596 old policies, ensuring that training stays within a stable trust region defined by  $\delta$ . Integrating the  
 597 components above, the loss function can be expressed as:

$$\sum_{\mathbf{v}_i \in \mathcal{V}} \mathbb{E}_{\mathbf{s}_i, \mathbf{a}_i \sim d^{\pi_{\theta_{old}}}} \left[ \min \left( \frac{\pi_{\theta}(\mathbf{a}_i | \mathbf{s}_i)}{\pi_{\theta_{old}}(\mathbf{a}_i | \mathbf{s}_i)} A^{\pi_{old}}(\mathbf{s}_i, \mathbf{a}_i), \text{clip} \left( \frac{\pi_{\theta}(\mathbf{a}_i | \mathbf{s}_i)}{\pi_{\theta_{old}}(\mathbf{a}_i | \mathbf{s}_i)}, 1 - \lambda, 1 + \lambda \right) A^{\pi_{old}}(\mathbf{s}_i, \mathbf{a}_i) \right) \right] \quad (2)$$

598 where  $d^{\pi_{\theta_{old}}}$  represents the probability of encountering a state  $\mathbf{s}_i$  and performing an action  $\mathbf{a}_i$  while  
 599 following the old policy  $\theta_{old}$ , and  $\mathbb{E}$  is the expectation over  $d^{\pi_{\theta_{old}}}$  for all robots. Gradients computed  
 600 from this objective are used to train the individual robot navigation policy  $\pi_{\theta}$  at the low level, and  
 601 backpropagated to the intermediate level to update weights  $\mathbf{W}^f$  of the graph network  $\phi$  for formation  
 602 adaptation.

603 **B.2 STAF Execution**

604 During execution, STAF performs centralized planning with decentralized execution. STAF assumes  
605 the 2D occupancy map of the environment along with the starting and goal positions of the robots as  
606 input, just as in training. The deep graph cut at the high level of STAF is performed in a centralized  
607 manner (although decentralization is possible, as discussed in Section 7): Each robot broadcasts its  
608 state via wireless communication (e.g., Wi-Fi), and a designated robot collects the states from all its  
609 teammates to compute the subteam assignments. After subteam division, formation adaptation at  
610 STAF’s intermediate level is performed in a decentralized manner. Through the same broadcasting  
611 mechanism via Wi-Fi, each robot can determine the relative positions of its teammates, which enables  
612 the robot to dynamically adjust its own position in relation to others to maintain and adapt the  
613 designated formation. Then, each robot executes velocity commands derived from the individual  
614 robot control policy at the low level of STAF.

615 **B.3 STAF Time Complexity Analysis**

616 Training time complexity is dominated by  $O(n^2)$ , where  $n$  is the number of robots. The high level  
617 has an  $O(HL_hT_hDn^2)$  complexity, where  $H$  is the number of attention heads of the transformer  
618 encoder with  $L_h$  layers in the upper-level GNN,  $T_h$  is the number of graph training epochs, and  $D$  is  
619 the number of samples for network training. The intermediate level has an  $O(L_mT_pn^2)$  complexity,  
620 where  $L_m$  is the number of layers in GNN, and  $T_p$  is the number of training iterations using PPO.  
621 The low-level training has an  $O(T_p(Bn^2 + IBn))$  complexity, where  $B$  is the number of PPO’s  
622 rollouts to interact with the environment in each iteration, and  $I$  is the number of PPO training  
623 epochs.  $O(Bn^2)$  is for computing the advantage function and  $O(IBn)$  is for updating the policy.  
624 Combining all terms, the overall complexity for training is  $O(HL_hT_hDn^2 + L_mT_pn^2 + T_p(Bn^2 +$   
625  $IBn))$ . Execution time complexity is dominated by  $O(n^2)$ . The complexities of the three levels  
626 are  $O(HL_hn^2)$ ,  $O(L_mn^2)$ , and  $O(n)$ , respectively. Thus, the overall execution complexity is  
627  $O(HL_hn^2 + L_mn^2 + n)$ . We will add a new subsection in the approach section to discuss details of  
628 the time complexities.

629 **C Experiment Setup**

630 To implement STAF, the edges in the robot team graph are constructed by connecting the nearby  
631 robots within a radius setting to 2 meters. STAF’s high-level deep graph cut network contains  
632 one linear layer with  $\mathbf{W}^v$  setting to the dimension of  $2 \times 32$  and three transformer layers with  
633 the parameter  $\mathbf{W}^a$  and  $\mathbf{W}^h$  setting to the dimension of  $32 \times 32$ . The intermediate-level GNN for  
634 formation adaptation contains one encoder with  $\mathbf{W}^z$  setting to the dimension of  $6 \times 64$  and one  
635 GNN layer with  $\mathbf{W}^g$  setting to the dimension of  $64 \times 64$ . The hyper-parameter  $\lambda = 0.6$  in the  
636 spring-damper model of STAF is to balance spring and damper force.

637 We generate synthetic data to train our STAF approach. Specifically, given a robot team formation,  
638 we randomly generate the positions of the robots within the formation. In total, we collect 10,000  
639 data instances to train our high-level network. The high-level network is trained for 100 epochs, while  
640 the adaptive formation control policy, involving both the intermediate-level and low-level neural  
641 networks, is trained over a total of 800 epochs. This alternating training of the high-level and joint  
642 intermediate-low-level networks continues until convergence.

643 **D Examples of Computing the CFI Evaluation Metric**

To evaluate formation adaptation, we introduce Contextual Formation Integrity (**CFI**) metric in our  
paper, which is mathematically defined as:

$$w(1 - \sigma^{-1} \min(|r - (\eta + \sigma)|, |r - (\eta - \sigma)|)) + (1 - w)\epsilon$$

644 where the first term assesses the team’s efficiency in utilizing the corridor gap, where  $r$  is the robot  
645 team’s maximum radius,  $\eta$  denotes the corridor width with a safety margin, and  $\sigma$  is a threshold with

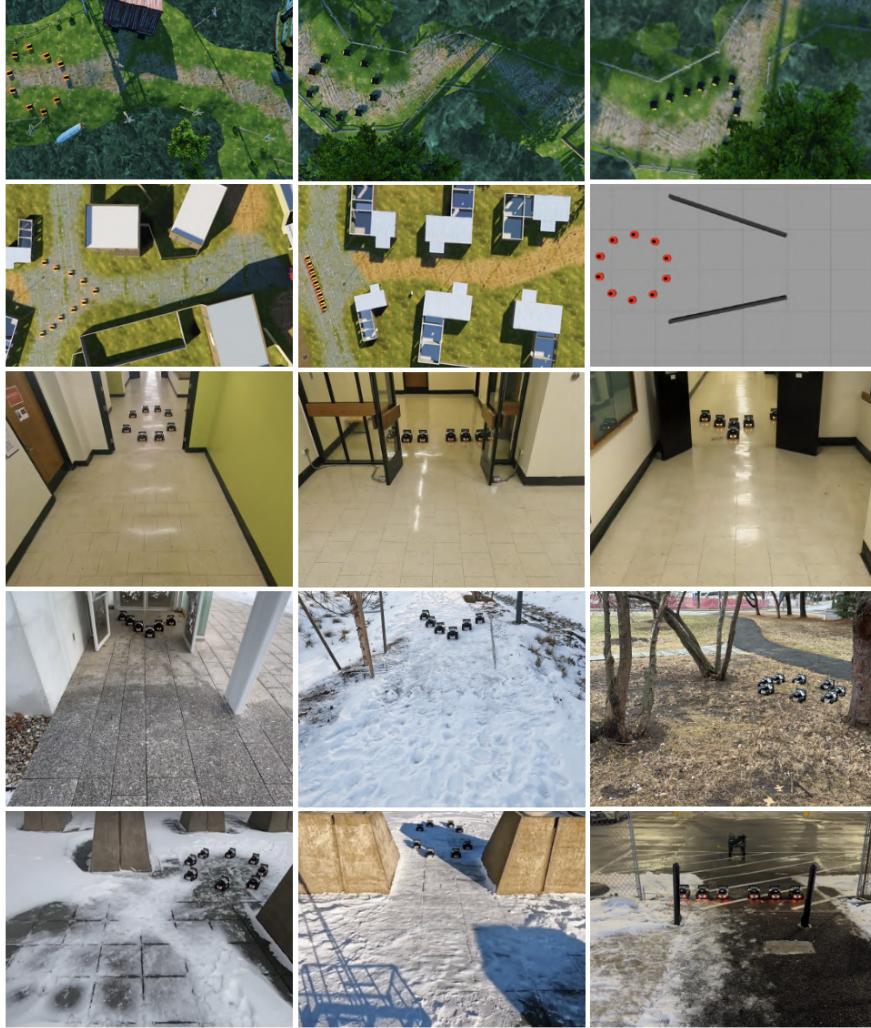
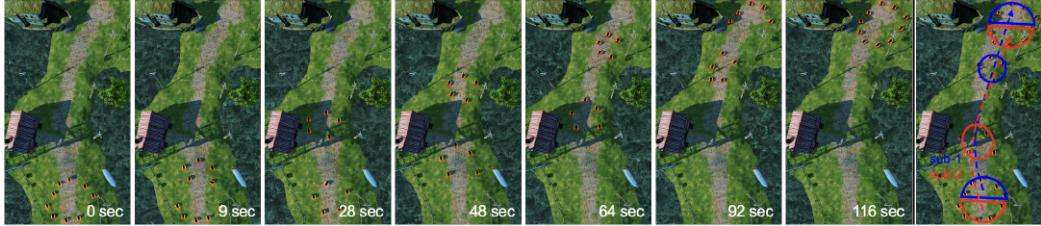


Figure 10: Example scenarios used to comprehensively evaluate and validate out STAF approach in multi-robot simulations as well as using real physical robots in both indoor and outdoor environments.

646 smaller values imposing stricter formation requirements. CFI's second term  $\epsilon \in [0, 1]$  evaluates the  
 647 integrity of the team shape. CFI combines these two terms to evaluate how effectively a robot team  
 648 uses the corridor space and maintains its formation, with the balance determined by the coefficient  $w$ .  
 649 The metric CFI  $\in [0, 1]$ , where higher values indicate better performance. In our experiments, we set  
 650  $w = 0.5$  to treat the gap usage and the formation integrity equally important. Additionally, we set  $\sigma$   
 651 to twice the width of the robot used in the corresponding experiments. For a number of  $n$  robots, the  
 652  $\epsilon$  in CFI is computed as follows:

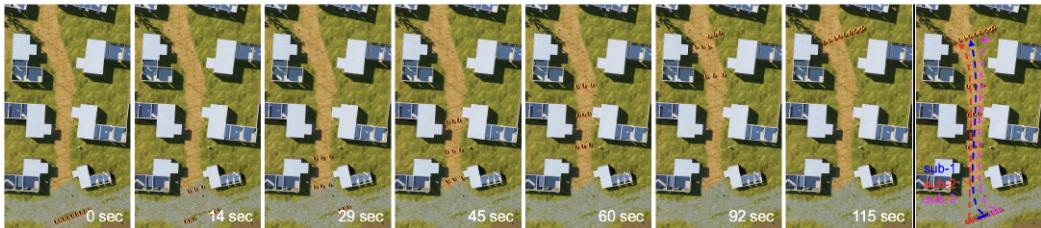
- 653 • Circle formation:  $\epsilon = 1 - \frac{1}{n} \sum_{i=1}^n \frac{\theta_i}{\frac{(n-2) \times 180}{n}}$ , where  $\theta_i$  represents the interior angle of the  
 654 triangle with the  $i$ -th robot as the vertex, and  $\frac{(n-2) \times 180}{n}$  is the interior angle of the polygon,  
 655 approximating a circle when the team has  $n$  robots.
- 656 • Wedge Formation:  $\epsilon = 1 - \frac{2|L_l - L_r|}{L_l + L_r} - \frac{|2L_m - L_b|}{L_b}$ , where  $L_l, L_r, L_m, L_b$  represent the lengths  
 657 of the left, right, middle, and base sides of the isosceles triangle formed by the robots.
- 658 • Line Formation:  $\epsilon = 1 - \frac{1}{n-1} \sum_{i=1}^{n-1} \frac{L_{i,i+1}}{L}$ , where  $L_{i,i+1}$  represents the distance between  
 659 neighboring robots, and  $L$  denotes the full width of the robot team. The term  $\epsilon$  measures the  
 660 relative deviation from the ideal line formation.



(a) A team of ten Warthog robots with a circle formation navigates through a narrow corridor on uneven terrain.



(b) A team of ten robots in a wedge formation traverses a progressively narrowing corridor between buildings.



(c) A team of nine robots in a line formation navigates through multiple narrow passages.

Figure 11: Qualitative results on subteaming and formation adaptation during coordinated multi-robot navigation using a high-fidelity Unity3D simulations in ROS1. The experiments adopt different numbers of differential-drive Warthog robots that maintain circle, wedge and line formations while traversing an unstructured outdoor field environment

## 661 E Extended Experiments

662 We comprehensively evaluate our STAF approach across diverse scenarios, with several representative  
 663 examples shown in Figure 10. Due to space limitations, additional qualitative results with extended  
 664 timesteps from Unity3D simulations, and real-world indoor and outdoor experiments are provided in  
 665 the appendix, highlighting the progression of subteaming and formation adaptation over time.

666 **Case Studies on High-fidelity Unity 3D Simulations** As illustrated in Figures 11(a) and 11(b), our  
 667 STAF approach successfully divides the full team into subteams and smoothly adjusts the actions of  
 668 differential-drive robots to navigate complex and curved trajectories toward the goal. This validates  
 669 the effectiveness of our method in handling both formation adaptation and subteaming in challenging  
 670 environments. Figure 11(b) specifically demonstrates subteaming and formation adaptation in a  
 671 wedge formation while navigating a progressively narrowing corridor between buildings. Our subteam  
 672 division emphasizes subteam-goal distance, resulting in more compact subteam formations, as shown  
 673 at 8 seconds. By 46 seconds, the formation adaptation capability of the two subteams becomes clearly  
 674 evident. In addition, Figure 11(c) shows that our approach dynamically divides the team into three  
 675 subteams, enabling successful navigation through the narrow corridor and subsequent regrouping.  
 676 This further demonstrates the capability of STAF to handle subteaming beyond two subteams in  
 677 extremely constrained scenarios.

678 **Case Study Validation on Physical Robot Teams** We validate our STAF method through case  
 679 studies involving real physical multi-robot teams, using differential-drive Limo robots equipped



(a) A team of eight physical robots with a circular formation navigates through a narrow doorway in a hallway.



(b) A team of six robots with a wedge formation navigates through a narrow exit.



(c) A team of six robots in a line formation navigates through a slightly wider but still confined doorway.

Figure 12: Qualitative results of coordinated multi-robot navigation, including team division, formation adaptation, and team recovery, using varying numbers of differential-drive Limo robots that maintain circle, wedge, and line formations across different indoor scenarios.

680 with caterpillar tracks. The experiments are conducted in both real-world indoor and outdoor  
 681 environments, and we present six representative scenarios in the paper, each highlighting various  
 682 real-world challenges. The indoor environments involve navigating constrained spaces, such as a  
 683 narrow doorway in a hallway, a tight exit from an indoor area to a partially open outdoor space, and a  
 684 slightly wider but still confined corridor. The outdoor experiments are conducted on unstructured  
 685 terrain, including a narrow passage between two concrete security bollards, a forest-like environment  
 686 with narrow corridors surrounded by scattered trees and obstacles, and a pathway with boundaries  
 687 marked by two sticks blocking vehicle access.

688 The experimental results using real robot teams in indoor environments are shown in Figure 12(a).  
 689 Our approach allows 8 Limo robots to dynamically divide into 2 subteams and successfully navigate  
 690 through a narrow doorway with formation adaptation. In the scenarios of narrow hallway and tight  
 691 exit, as shown in Figures 12(b) and 12(c), our approach continues to effectively facilitate subteaming  
 692 and formation adaptation within a robot team with 6 robots, ensuring smooth navigation through  
 693 constrained spaces in the real world. The experimental results using Limo robot teams in outdoor  
 694 environments are shown in Figures 13(a), 13(b) and 13(c). The results indicate a strong adaptation  
 695 capability of our approach to unknown environments; subteaming and formation adaptation can well  
 696 be performed on snowy and uneven terrain, where wheel slippage poses large action uncertainty. By  
 697 effectively coordinating robots within a team or subteam, our method achieves stable and adaptive  
 698 navigation, ensuring efficient team coordination even in highly uncertain and unknown environments.

## 699 F Extended Discussion

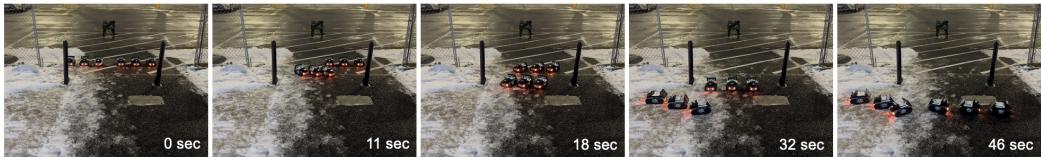
700 In the main paper, we analyze the characteristics of our STAF approach, focusing on its general-  
 701 ization to different team sizes and numbers of robots, and include an ablation study on subteam  
 702 division. Here, we demonstrate the approach’s robustness to noise and its applicability to different  
 703 robot platforms. While circle formations are included to highlight the characteristics of our approach,  
 704 we have conducted additional experiments using other formations and observed similar results.



(a) A team of eight physical Limo robots with a circle formation traverses a narrow passage between two concrete security bollards.



(b) A team of six robots in a wedge formation navigates through a forest-like environment with narrow corridors surrounded by scattered trees and obstacles.



(c) A team of six Limo robots in a line formation navigates through a narrow pathway with boundaries marked by two sticks blocking vehicle access.

Figure 13: Qualitative results of team division, formation adaptation, and team recovery during coordinated navigation using varying numbers of differential-drive Limo robots that maintain circle, wedge, and line formations across different unstructured outdoor environments.

#### 705 Robustness in Subteam Division to Noise

706 In order to analyze STAF’s robustness to noise  
 707 in subteam division, we first present the graph  
 708 cut performance in Figure 14(a) under normal  
 709 conditions. These conditions are defined as the  
 710 experimental setups where robot positions are  
 711 uniformly distributed along a circular edge, with  
 712 no noise introduced. Our STAF approach clearly  
 713 achieves an even division of the robot team into  
 714 two subteams, ensuring maximum adjacency  
 715 within each subteam and minimum distance be-  
 716 tween subteams and their respective goals. Then, to  
 717 simulate noise in robot state estimation, which is  
 718 often modeled as Gaussian [69], we add standard Gaussian noise to the robot positions, as illustrated  
 719 in Figure 14(b). Despite the added noise, our approach preserves a consistent subteam division, which  
 indicates the robustness of our STAF approach against positional perturbations.

#### 720 Applicability to Different Robot Platforms

721 We further demonstrate the applicability of our  
 722 STAF approach to different robot types. In high-  
 723 fidelity Unity3D simulation in ROS1, we test it  
 724 on differential-drive Warthog robots, while real-  
 725 world experiments involve Limo robots. Ad-  
 726 ditionally, we assess its performance with 10  
 727 holonomic-drive robots. As illustrated in Figure  
 728 15, our STAF approach successfully enables a  
 729 new team of holonomic robots to perform sub-  
 730 teaming and adaptive formation control to navigate through narrow corridors, with the support of an  
 731 external tracking and state estimation system using OptiTrack.

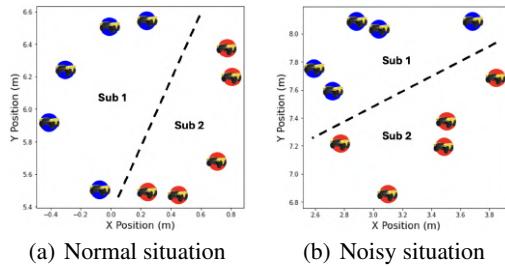


Figure 14: STAF’s robustness in graph-cut-based subteam division to noise.

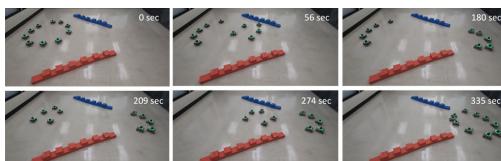


Figure 15: Applicability of STAF to a team of holonomic robots for coordinated navigation, supported by an external tracking system (OptiTrack).